Problem Solving in Physics and its Application to Collaborative Problem Solving
A Guide for PHYS 102

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Chapter 1

Introduction

A common perception of physics is that it is a collection of equations to be memorized, and that to successfully solve problems, all one needs to do is: find the right equation, plug in the numbers, and get the answer. This could not be further from the truth. Physics is a collection of ideas that are represented mathematically. The reason for the mathematical representation is so that we can test our ideas against experiment. A good way to think of the relationship between physics and mathematics is to think of mathematics as the language of physics. In a language, each sentence is made up of many different words. Each of these words has an individual meaning and serves a particular function in the sentence. In physics, the goal is to write a sentence (equation) describing a desired relationship between all of your data (variables). Thus, you use mathematics to *describe* your understanding of the *physical concepts* at work. Moreover, just as writing sentences helps you improve your proficiency in a language, solving physics problems can help deepen your understanding of the concepts at work.

A reflection of this incorrect perception of physics are the two common statements heard in office hours, “I understand the concepts, but I cannot do the problems.” and “I can follow the examples in class and in the textbook but your problems are just too different.” Commonly, such statements are a reflection of more fundamental issues. Two of the most common are a lack of a coherent conceptual framework (Heller and Heller, 2010) and/or a lack of logical analysis.

A coherent conceptual framework implies a mental map showing the different ideas of just linear kinematics and linear dynamics and how they fit together. Figure 1.1 shows a *rough* map that I made on my whiteboard to determine the topic order for PHYS 102. You can see that there are many ideas and that they connect in many different ways.

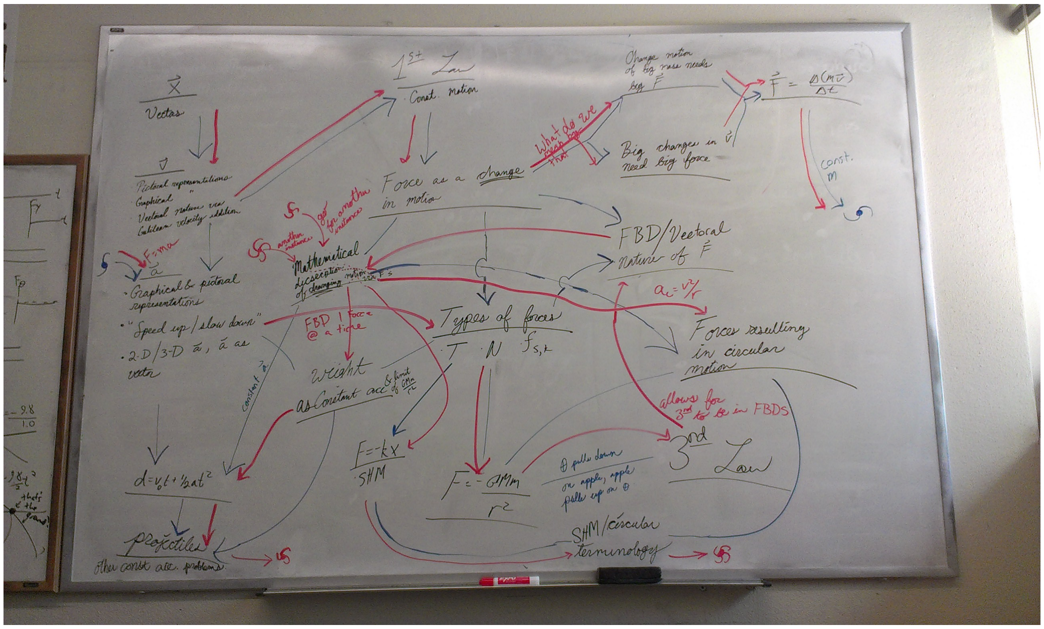

Figure 1.1: A mental map of linear kinematics and dynamics that I made on a whiteboard to prepare for PHYS 102. All the connections make it quite messy!

Logical analysis is the skill of applying conceptual knowledge to a new problem in a way that is likely to yield a solution: exactly the situation under which you are going to be tested! Furthermore, this is probably the most important skill that you can take away from this class. I know that most of you will never solve another physics problem again once you leave the University. However, as people in the scientific and technical fields, you will be encountering new problems every day. Being able to methodically approach them is critical.

This workbook is designed to provide resources to help you learn to solve physics problems more effectively. Much of the information comes from Heller and Heller (2010). Chapter 2 contains an overview of what a problem solving framework is from David DeMuth (2014). Chapter 3 contains an outline of a problem solving strategy that we will use in this class (Heller and Heller, 2010). Chapter 4 discusses a worksheet, found in Appendix A modeled on the framework that you will use for in-class problem-solving activities. During these in-class problem-solving activities, you will be working in groups of three or four. Chapter 5 contains information about how these groups will be structured.

Chapter 2

What is a Problem Solving Framework?

The following text is predominately from David DeMuth (2014)

2.1 Introduction

At one level, problem solving is just that, solving problems. Presented with a problem you try to solve it. If you have seen the problem before and you already know its solution, you can solve the problem by recall. Solving physics problems is not very different from solving any kind of problem. In your personal and professional life, however, you will encounter new and complex problems. The skillful problem solver is able to invent good solutions for these new problem situations. But how does the skillful problem solver create a solution to a new problem? And how do you learn to be a more skillful problem solver?

Research in the nature of problem solving has been done in a variety of disciplines such as physics, medical diagnosis, engineering, project design and computer programming. There are many similarities in the way experts in these disciplines solve problems. The most important result is that experts follow a general strategy for solving all complex problems. If you practice and learn this general strategy you will be successful in this course. In addition, you will become familiar with a general strategy fro solving problems that will be useful in your chosen profession.

2.2 A Logical Problem-Solving Strategy

Experts solve real problems in several steps. Getting started is the most difficult step. In the first and most important step, you must accurately visualize the situation, identify the actual problem , and comprehend the problem . At first you must deal with both the qualitative and quantitative aspects of the problem. You must interpret the problem in light of your own knowledge and experience. This enables you to decide what information is important, what information can be ignored, and what additional information may be needed, even if it was not explicitly provided. In this step it is also important to draw a picture of the problem situation. A picture is worth a thousand words if, of course, it is the right picture. (If a picture is worth a thousand words, and words are a dime a dozen, then what is a picture's monetary value?) In the second step, you must represent the problem in terms of formal concepts and principles, whether these are concepts of architectural design, concepts of medicine, or concepts of physics. These formal concepts and principles enable you to simplify a complex problem to its essential parts, making the search for a solution easier. Third, you must use your representation of the problem to plan a solution. Planning results in an outline of the logical steps required to obtain a solution. In many cases the logical steps are conveniently expressed as mathematics. Fourth, you must determine a solution by actually executing the logical steps outlined in your plan. Finally, you must evaluate how well the solution resolves the original problem.

The general strategy can be summarized in terms of five steps:

1. Comprehend the problem.
2. Represent the problem in formal terms.
3. Plan a solution.
4. Execute the plan.
5. Interpret and evaluate the solution.

The strategy begins with the qualitative aspects of a problem and progresses toward the quantitative aspects of a problem. Each step uses information gathered in the previous step to translate the problem into more quantitative terms. These steps should make sense to you. You have probably used a similar strategy when you have solved problems before.

2.3 A Physics-Specific Strategy

Each profession has its own specialized knowledge and patterns of thought. The knowledge and thought processes that you use in each of the steps will depend on the discipline in which you operate. Taking into account the specific nature of physics, we choose to label and interpret the five steps of the general problem solving strategy as follows below. Note that the steps contain a lot of writing. This is due to limits of the human brain. The typical human brain can hold 7 ± 2 pieces of information in short-term working memory (George A. Miller, 1956). Most physics problems require more information. Thus, it is physiologically impossible to do all of this in your head without writing it down.

2.3.1 Focus the Problem

In this step you develop a qualitative description of the problem. First, visualize the events described in the problem using a sketch. Write down a simple statement of what you want to find out.

The next part of this step is perhaps against your expectations: write down the physics ideas which might be useful in the problem *in words*. Are you thinking about forces, energy, or both? Are things moving in straight lines or is there rotation in the problem? If the problem refers to a situation with which you have familiarity, what is going to happen? For example, I know that if the problem involves launching a ball from the surface of the earth, then the ball will go up and then come back down. Do not use equations in this step. Using words will help you become more familiar with the language of physics and help you develop an understanding of some of the more abstract quantities.

Finally, think about what approximations and constraints, if any, you need to make. Often in physics we consider idealized situations which involve several approximations. We do these approximations for two reasons. First, by studying simple idealized situations we learn about the true rules of the Universe. Second, and more practically, it makes the problems do-able in a reasonable amount of time! Consciously think about what approximations are justified in this situation. Can certain effects be neglected? How do you know? Constraints are limits to how things can move. For example, if the problem refers to a car going around a circular race track, then we know that the car must be a set distance from the center of the circular track.

Consciously think about these constraints and write them down.

2.3.2 Describe the Physics

In this step you use your qualitative understanding of the problem to prepare for the quantitative solution. First, simplify the problem situation by describing it with a diagram in terms of simple physical objects and essential physical quantities. Restate what you want to find by naming specific mathematical variables. Using the physics ideas assembled in step 1, write down the equations which specify how these physical quantities are related according to the principles of physics or mathematics.

2.3.3 Plan the Solution

In this step, you need to translate the physics description into a set of equations which represent the problem mathematically by using the equations assembled in step 2. Write down an outline of how you will solve these equations to see if they will yield a solution before you go through the effort of actually doing any mathematics. Finally, make a very rough estimate of your result. If the problem asks for a distance, should your answer be on the scale of people (meters), countries (thousands of kilometers), or atoms (nanometers)? This will help you during execution of your plan; if you are solving a problem about atoms and you get distances of 10^4m, then you are probably doing something wrong!

2.3.4 Execute the Plan

In this step you actually execute the solution you have planned. Combine the equations as planned to first determine an algebraic solution. Then plug in all of the known quantities into the algebraic solution to determine a numerical value for the desired unknown (target) quantity.

2.3.5 Evaluate the Answer

Finally, check your work to see that it is properly stated, reasonable, and that you have actually answered the question asked.

2.4 Summary

Consider each step as a translation of the previous step into a slightly different language. You begin with the full complexity of real objects interacting in the real world and through a series of steps arrive at a simple and precise mathematical expression.

The five-step strategy represents an effective way to organize your thinking to produce a solution based on your best understanding of physics. The quality of the solution depends on the knowledge that you use in obtaining the solution. Your use of the strategy also makes it easier to look back through your solution to check for incorrect knowledge and assumptions. That makes it an important tool for learning physics. If you learn to use the strategy effectively, you will find it a valuable tool to use for solving new and complex problems. After all, those are the ones that you will be hired to solve in your chosen profession.

Chapter 3

Outline of the Framework

An outline of the problem solving framework that we are going to use can be found beginning on the next page. The outline is based upon work in Heller and Heller (2010) with a few modifications. You can use this page as a short-hand reminder of what the various steps in the framework are. I would recommend that you bring a copy to class every day as we will be using this framework extensively. Keep in mind, however, that a problem-solving framework is only a guide. It is not a linear sequence of steps to be followed. True problem solving usually involves looping around between and within steps and involves constantly asking yourself the questions, "what do I know, and how do I know it?"

I) **Focus the Problem:** Establish a clear mental image of the problem.

   A) Visualize the situation and events by sketching a useful picture

      1) Draw a picture showing how the objects are related spatially, how they are moving, and how they are interacting. The drawing should show the time sequence of events, especially for those times when an object experiences an abrupt change.

      2) Write down the relevant known and unknown information, giving each quantity a symbolic name and adding that information to the picture.

   B) Describe *in words* what is *physically happening* in the problem.

      1) Think in terms of our physics language. Are there forces acting in the problem? What is the source of the force? What object is feeling the force? Is there a net acceleration?

      2) "What is going to happen?" Will the car roll down the hill?

   C) Identify any constraints applicable to the problem.

      1) For example:

         a) A car must stay on the road

         b) If one block is pushing another, then the accelerations of the two blocks must be the same

      2) Some equations that we will encounter are only valid under certain conditions such as when objects are undergoing motion in a circle. This is where you should start thinking about these.

   D) Precisely state the question to be answered in terms you can calculate.

II) **Describe the physics**

A) Re-draw (or add to your existing drawing) any diagrams with more information such as coordinate systems, free-body diagrams, etc. that are consistent with the approach(es) you have chosen.

B) Define consistent and unique symbols for any quantities that are relevant to the situation.

C) Identify the target quantity(s) that will provide the answer to the question.

D) Only now start collecting equations. Assemble the appropriate equations to quantify the physics principles and constraints identified in your approach.

III) **Plan a solution**

A) Make a very rough estimate of your result. If the problem asks for a distance, should your answer be on the scale of people (meters), countries (thousands of kilometers), or atoms (nanometers)? In what direction is the result?

B) Construct a logical chain of equations from those identified in the previous step, leading from the target quantity to quantities that are known.

1) Follow your description of what is happening in words
2) Begin with the quantitative relationship that contains the target variable. Identify other unknowns in the equation.
3) Choose a new equation for one of these unknowns. Keep track of any additional unknowns.
4) Continue this process for each unknown.

C) Determine if this chain of equations is sufficient to solve for the target quantity by comparing the number of unknown quantities to the number of equations.

D) Write down a verbal description of the solution steps you will take to solve this chain of equations so that no algebraic loops are created. Work from the last equation to the first equation that contains the target quantity.

IV) **Execute the plan:** This is the first place you should start doing algebra

   A) Follow the outline from in the previous step.

   B) Arrive at an algebraic equation for your target quantity by following your verbal description of the solution steps.

   C) Check the units of your final algebraic equation before putting in numbers.

   D) If quantities have numerical values, substitute them in your final equation to calculate a value for the target quantity.

V) **Evaluate the answer**

A) Does the mathematical result answer the question with appropriate units?

B) Is the result unreasonable? Compare to your rough estimate done in "Describe the physics"

C) Is the answer complete? Does it answer the question asked?

Chapter 4

Problem solving worksheet

In this class you will be doing problems in groups during class. To help you practice problem solving, you will be expected to complete your solution on the problem solving worksheet which is in Appendix A. A copy of this worksheet will also be available on d2L. The worksheet structure will help you develop your skills in logical analysis. The worksheet is based upon work in Heller and Heller (2010), but I have made adjustments to target those issues that students in my classes have had in the past.

This worksheet can also be helpful when you are solving homework problems. "I don't know how to start the problem" is a common phrase heard in office hours. When you feel this way, try using one of these worksheets to help you get started!

Chapter 5

Working in Groups

Evidence clearly indicates that students who work in groups learn better than those who work individually (Heller and Heller, 2010). Every successful physics major the author has ever met has worked in groups. By working in groups you have the opportunity to discuss ideas and through that discussion reveal your own incorrect conceptions and develop a better understanding of the material. Moreover, almost no one in a scientific field works in isolation.

Research indicates that the ideal group size is three and that the most effective groups tend to be randomly assigned (Heller and Heller, 2010). To give you the opportunity to explore a variety of perspectives over the course of the semester, the groups will be shuffled every few weeks for a total of 3-5 times over the course of the semester.

To help with group efficiency and to ensure that your group thoroughly explores all aspects of the problem, each person in the group will have a defined role. These group roles are: Manager, Checker/Recorder, and Skeptic/Summarizer. The exact functions of each group member are detailed in Table 5.1. To solve a problem on your own successfully, you need perform all of these functions. The group roles will help you develop these reflection skills by allowing you to focus on one at at time.

In order to improve group effectiveness your group will, from time to time, be required to complete a group evaluation form such as that found in Appendix B. Research in Heller and Heller (2010) demonstrates that such group reflection improves the quality of the group experience for everyone. Such experiences may also help you personally become more effective/comfortable with working in groups in the future.

ACTIONS	WHAT IT SOUNDS LIKE*
MANAGER DIRECT THE SEQUENCE OF STEPS. KEEP YOUR GROUP "ON-TRACK." MAKE SURE EVERYONE IN YOUR GROUP PARTICIPATES. WATCH THE TIME SPENT ON EACH STEP.	*"First, we need to draw a picture of the situation."* *"Let's come back to this later if we have time."* *"Chris, what do you think about this idea?"* *"We only have 5 minutes left. Let's finish the algebraic solution.*
RECORDER/CHECKER ACT AS A SCRIBE FOR YOUR GROUP. CHECK FOR UNDERSTANDING OF ALL MEMBERS. MAKE SURE ALL MEMBERS OF YOUR GROUP AGREE WITH EACH THING YOU WRITE. MAKE SURE NAMES ARE ON SOLUTION.	*"Do we all understand this diagram I just finished?"* *"Explain why you think that . . . ."* *"Are we in agreement on this?"* *"Here, sign the problem we just finished!"*
SKEPTIC/SUMMARIZER HELP YOUR GROUP AVOID COMING TO AGREEMENT TOO QUICKLY. MAKE SURE ALL POSSIBILITIES ARE EXPLORED. SUGGEST ALTERNATIVE IDEAS. SUMMARIZE (RESTATE) YOUR GROUP'S DISCUSSION AND CONCLUSIONS. KEEP TRACK OF DIFFERENT POSITIONS OF GROUP MEMBERS AND SUMMARIZE BEFORE DECIDING.	*"What other possibilities are there for ...?"* *"I'm not sure we're on the right track here. Let's try to look at this another way. . . ."* *"Why?"* *"What about using . . . . instead of . . . . ?* *"So here's what we've decided so far."* *"Chris thinks we should . . . . , while Pat thinks we should . . . ."*

Table 5.1: The details of each group role

Appendix A

Group Worksheet

The following pages have the worksheet that you will use in class for problem solving. A copy will also be posted on d2L.

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

Focus on the Problem:

Sketch a useful picture(s):

Describe *in words* what is physically going on in this situation. Use physics terms such as "force" and "energy"

Describe *in words* any approximations or constraints (“the block must remain on the floor”):

Precisely state the question in terms of quantites you can calculate:

Describe the Physics:

Add to your drawing(s) any diagrams needed that are consistent with your approach: (coordinate systems/vectors/FBDs etc.)

Define unique and consistent symbols for all relevant quantities. Circle the one you are looking for.

Based upon your descriptions of the physics involved, list those equations that may/will be relevant (physics and constraints!). Cross out terms that are zero. For example, if you are thinking about forces, F=ma would go here.

Plan a Solution:

Make a *rough* estimate of your result. Can you figure out the direction at this point?

Using your written description of what is going on, develop logical chain of equations:

Check for sufficiency: ☐

Execute the Plan:

Algebraic manipulations ONLY:

Unit check of final equation:

Substitute numbers:

Evaluate Solution:

Unit check ☐

Is your answer reasonable? How do you know?

Completeness check ☐

Appendix B

Group Performance Evaluation Worksheet

The following pages have the worksheet that you will use in class for evaluating your groups performance. A copy will also be posted on d2L.

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

Name: ______________________________ Group Role: ______________________________

1. Use the following grid to rate yourself on your participation and contributions in your group's problem solving. Also agree on a group rating: **0 = Poor, 1 = Fair, 2 = Good, 3 = Excellent**. Use unique initials

	Initials:	Initials	Initials	Initials:	Group
Participation in solving problem					
Contribution of ideas to the thorough analysis *before* generating appropriate equations					
Contribution of ideas to *planning* the mathematical solution					
Overall use of a logical, organized approach to solving the problem					

2. What are two **specific** actions we did today that helped us work together towards a successful solution?

3. What is a **specific** action that would help us do even better next time?

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Bibliography

David DeMuth (2014) Minnesota model - the Minnesota problem solving strategy. URL http://groups.physics.umn.edu/physed/Research/CRP/psintro.html

George A Miller (1956) The magical number seven, plus or minus two: some limits on our capacity for processing information. Psychological Review 63(2):81–97, DOI 10.1037/h0043158

Heller K, Heller P (2010) Cooperative problem solving in physics a users manual: Why? what? how? In: AAPT New Faculty Conference, URL http://www.aapt.org/Conferences/newfaculty/upload/Coop-Problem-Solving-Guide.pdf